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1994 J. Phys.: Condens. Matter 6 7547

(http://iopscience.iop.org/0953-8984/6/37/009)

J. Phys.: Condens. Matter 6 (1994) 7547–7552. Printed in the UK

Dynamical properties of weakly linked, mesoscopic, superconducting dots

P M A Cook, A Martin and C J Lambert
School of Physics and Materials, Lancaster University, Lancaster LA1 4YB, UK

Received 26 April 1994, in final form 13 June 1994

Abstract. We compute the current–voltage (I–V) characteristic of a mesoscopic Josephson junction, formed from two weakly linked, superconducting dots. A key feature of such structures is the presence of quasi-particle resonances in the current–phase relation. We show that these lead to measurable changes in the I–V characteristic compared with conventional macroscopic junctions. For an overdamped junction, the I–V characteristic is predicted to shift to higher values of the current, but the critical current is unchanged. For an underdamped junction exhibiting hysteresis, this shift is accompanied by an increase in the critical current associated with the lower branch of the I–V curve.

1. Introduction

During the past decade [1, 2] phase coherent transport in mesoscopic structures has formed a subject of considerable interest. In such systems, the sample size is smaller than the inelastic phase breaking length l_ϕ and transport properties are determined by interference patterns formed from electronic wavefunctions near the Fermi energy. While this interest was originally focused on normal systems, attention has now turned towards hybrid structures, obtained by placing one or more superconductors in contact with a mesoscopic host. One example of such structures is a mesoscopic Josephson junction formed when a weak link connecting two superconductors is smaller than l_ϕ. If the superconductors are longer than l_ϕ, only quasi-particles within the link are phase coherent and the theory of [3, 4] is applicable. On the other hand, if the superconductors take the form of mesoscopic dots of size less than l_ϕ, quasi-particles can resonantly tunnel into the weak link from external reservoirs, leading to significant changes in the current–phase relation of the device [5]. In this paper, with a view to identifying experimental signatures of the transition to mesoscopic dynamics, we examine the effect of resonances on the I–V characteristic of such linked mesoscopic (LM) dots. Since no tractable, microscopic theory of the dynamical behaviour of LM dots currently exists, we adopt a somewhat heuristic approach by examining the change in the I–V characteristic of a resistively shunted junction, produced by the presence of quasi-particle resonances in the current–phase relation.

2. The model

In [5] a scattering theory approach was used to examine transport through a pair of LM dots described by the Bogoliubov–de Gennes equation

$$\mathsf{H}(x)\Psi(x) = E\Psi(x) \tag{1}$$

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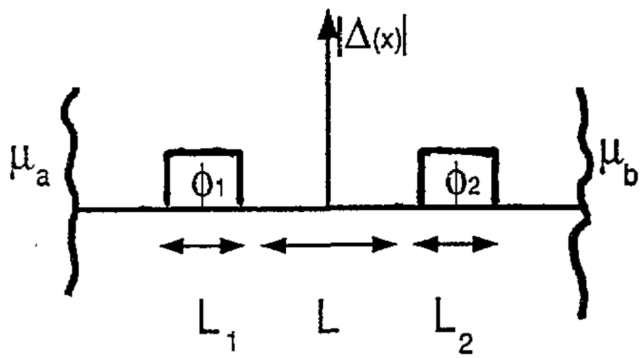

Figure 1. A simple model of a pair of LM dots. This figure shows an order parameter with a non-zero value in the regions occupied by the dots and possessing a phase difference $\phi = \phi_1 - \phi_2$. The structure is connected to external reservoirs at chemical potentials μ_a and μ_b.

where

$$\mathsf{H} = \begin{pmatrix} [-(\hbar^2/2m)\partial_x^2 + u(x) - \mu] & \Delta(x) \\ \Delta^*(x) & -[-(\hbar^2/2m)\partial_x^2 + u(x) - \mu] \end{pmatrix}. \tag{2}$$

In this equation, μ is the condensate chemical potential, $u(x)$ the normal scattering potential and $\Delta(x)$ the superconducting order parameter. A simple model of a pair of LM dots, shown in figure 1, is obtained by allowing $\Delta(x)$ and $u(x)$ to be non-zero only in regions of size L_1 and L_2, where the order parameter phase takes values ϕ_1 and ϕ_2 respectively. Figure 1 shows a pair of LM dots connected by perfect, normal leads, to external reservoirs at chemical potentials μ_a and μ_b. In conventional descriptions of the Josephson effect [6, 7], the sources of charge are not incorporated at a quantum mechanical level and the lengths L_1 and L_2 are taken to be infinite. For LM dots, the system size $L_1 + L + L_2$ is assumed to be smaller than the quasi-particle phase breaking length and therefore a description that incorporates quasi-particle phase coherence throughout the whole structure is appropriate. In [5] by computing the quantum mechanical scattering matrix for the structure shown in figure 1, it was predicted that quasi-particles from external reservoirs can resonantly tunnel into the weak link and produce a current–phase relation which differs markedly from the conventional sinusoidal form. While the detailed form of this relation depends on the precise shape of the potentials $u(x)$, $\Delta(x)$, a key prediction is the presence of a new quasi-particle peak, which is missing from non-mesoscopic structures. For simplicity, to capture the new generic features associated with such resonances, we model the current–phase relation predicted in [5] by the function

$$\begin{aligned} f(\phi) &= 1 + \sin(\phi) \qquad & -\pi \leqslant \phi < -\pi + \delta \\ f(\phi) &= \sin(\phi) \qquad & -\pi + \delta \leqslant \phi < \pi. \end{aligned} \tag{3}$$

In this equation, $f(\phi)$ is periodic in 2π and $\delta \geqslant 0$ is a symmetry breaking parameter arising from the presence of external quasi-particle reservoirs. Equation (3) contains the essential features predicted for LM dots [5], namely a quasi-particle peak of width δ, superimposed on an odd function of the phase. To examine how the I–V characteristic changes from its conventional form as δ increases from zero, we insert this into the standard model of a resistively shunted junction [6, 8]

$$\ddot{\phi} + \gamma\dot{\phi} + F(\phi) = I. \tag{4}$$

In this equation, following [5], we have written $F(\phi) = sf(s\phi)$, where s is the sign of I. Even though $F(\phi)$ is not an odd function of ϕ, the I–V characteristic is an odd function of V and therefore in what follows, only the case $s = 1$, $V \geqslant 0$ will be examined.

3. Results

Figures 2 and 3 show results for overdamped ($\gamma > 1$) and underdamped $\gamma < 1$ junctions respectively, obtained by numerically integrating equation (4). The figures show results for the externally supplied current I against the voltage $V = \langle\dot{\phi}\rangle$, where $\langle\rangle$ denotes a time average. Figure 2 shows that for overdamped systems, the introduction of a quasi-particle peak shifts the I–V characteristic to lower values of V. For underdamped systems, figure 3 shows that a larger shift occurs and in addition, the critical current is increased by increasing the resonance width δ. More precisely, consider first the three $\delta = 0$ curves of figure 3, corresponding to γ =1, 0.5, 0.2. As predicted by Stewart [8], these underdamped junctions exhibit hysteresis: with increasing current, a discontinuous jump to a finite voltage occurs at a critical current $I_c^+ = 1$, independent of δ, whereas with decreasing current, a discontinuous jump to $V = 0$ occurs at a smaller critical current $I_c^-(\gamma)$. For example figure 3 shows that for $\gamma = 0.2$ and $\delta = 0$, $I_c^- \simeq 0.25$ and furthermore $I_c^-(\gamma)$ increases with increasing δ. Moreover, for $\delta \neq 0$, an extrapolation of the ohmic portion of the characteristic does not pass through the origin.

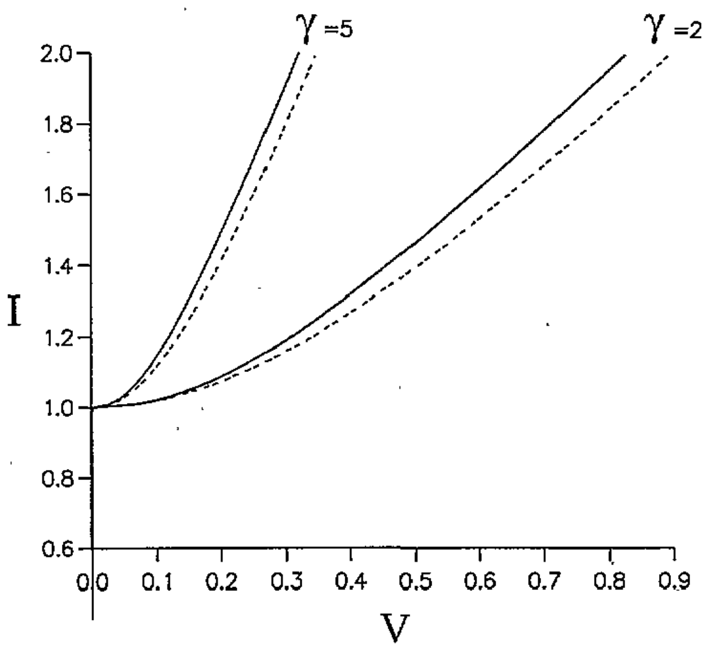

Figure 2. Current–voltage characteristics for overdamped junctions, with $\gamma = 2$ and $\gamma = 5$. The dashed line correspond to $\delta = 0$, while the solid lines correspond to ($\delta = 1.14$).

To gain some understanding of this behaviour, we generalize the analysis of [8] to the case $\delta \neq 0$. Since in steady state, the time-averaged acceleration must vanish, equations (2)

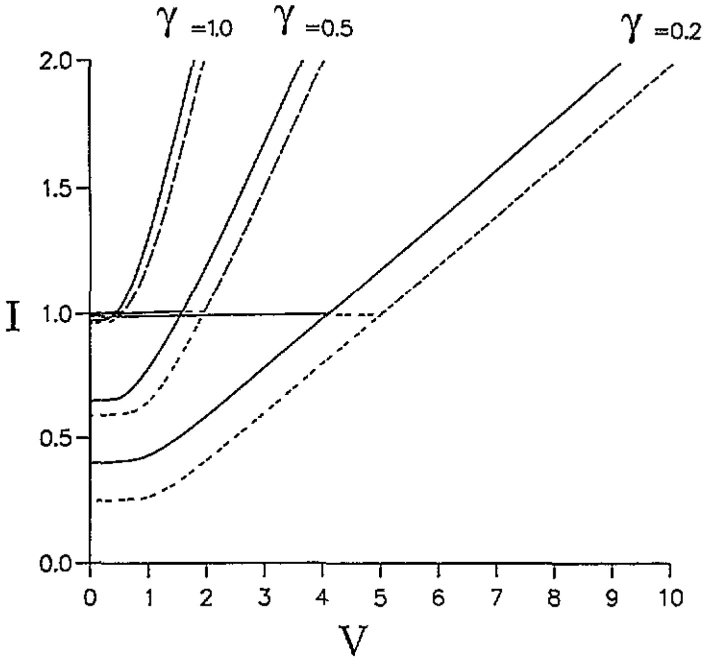

Figure 3. Current–voltage characteristics for underdamped junctions. The dashed lines correspond to $\delta = 0$, while the solid lines correspond to $\delta = 1.14$. With decreasing current, both systems exhibit a discontinuous jump to $V = 0$, although since results are obtained by carrying out simulations at a closely spaced, but finite number of points, the lines intersecting the $V = 0$ axis are not quite horizontal.

and (3) yield $\gamma\langle\dot{\phi}\rangle = I - \delta/2\pi$. To lowest order in γ, as I decreases towards I_c^-, $\langle\dot{\phi}\rangle$ can be approximated by its value on the separatrix of the system with no damping, to yield

$$I_c^- = \delta/2\pi + (\gamma/2\pi)\int_{-\pi}^{\pi} \mathrm{d}\phi\,\dot{\phi}. \tag{5}$$

To obtain $\dot{\phi}$, we note that in the absence of damping, equation (4) becomes $\ddot{\phi} = -\partial_\phi U(I, \phi)$, where

$$\begin{aligned} U(I,\phi) &= -I\phi + \phi - \cos(\phi) + n\delta - (2n-1)\pi && (2n-1)\pi \leqslant \phi < (2n-1)\pi + \delta \\ U(I,\phi) &= -I\phi - \cos(\phi) + (n+1)\delta - 2n\pi && (2n-1)\pi + \delta \leqslant \phi < (2n+1)\pi. \end{aligned} \tag{6}$$

For a non-mesoscopic junction, with $\delta = 0$, this is a well known tilted, washboard potential. For $\delta \neq 0$, it takes the form shown in figures 4(a) ($I = 0$) and (b) ($I = 0.7$), which show that sharp changes occur in the slope of $U(I, \phi)$, due to the presence of resonances. For the case $\delta = 0$, the dynamical properties of this potential are well understood in terms of vibrational and rotational solutions and if the current is increased sufficiently, then only rotational solutions are possible.

In the absence of damping, the motion is obtained by noting that the energy $E = \dot{\phi}^2/2 + U(I, \phi)$ is a constant of the motion. On the separatrix, $E = U_0(I)$, where $U_0(I)$ is the maximum value of the potential in the interval $-\pi < \phi < \pi$. Since we are interested in

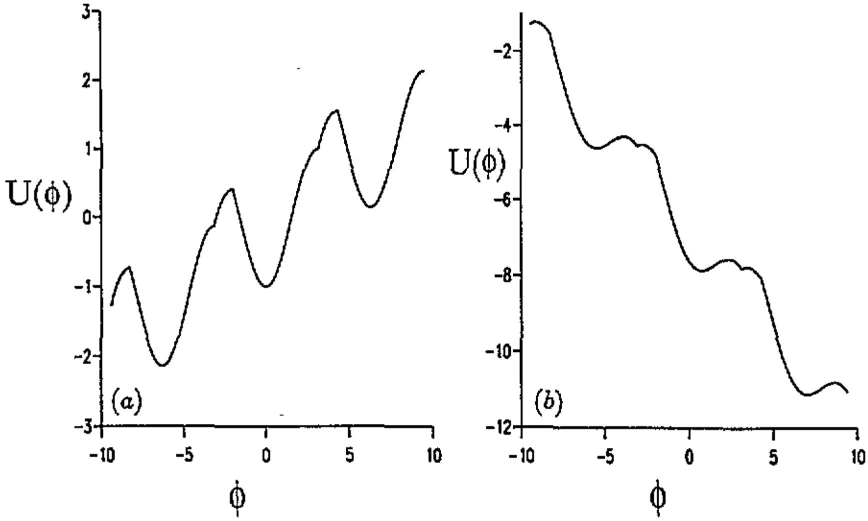

Figure 4. Graphs of the potential $U(I, \phi)$ given by equation (6). Figure 4(a) corresponds to $I = 0$ and figure 4(b) to $I = 0.7$.

the case $I = I_c^-$ and to zeroth order in γ, $I_c^- = \delta/2\pi$, the potential defining the separatrix is $U(\delta/2\pi, \phi)$, which has a maximum value of $U_0(\delta) = (1-\delta/2\pi)\phi_0 - \cos\phi_0 + n\delta - (2n-1)\pi$. In this equation, $\phi_0 = -\pi + \delta$ for $\delta < \delta_0$ and $\phi_0 = -\pi + \sin^{-1}(1-\delta/2\pi)$ for $\delta > \delta_0$, where $\delta_0 = 0.9991\ldots$ is the solution of $\sin^{-1}(1-\delta_0/2\pi) = \delta_0$. This yields, to lowest order in γ,

$$I_c^- = \delta/2\pi + (\sqrt{2}\gamma/2\pi)\int_{-\pi}^{-\pi+\delta} \mathrm{d}\phi\,[U_0(\delta) - (1-\delta/2\pi)\phi - \pi + \cos\phi]^{1/2}$$
$$+ (\sqrt{2}\gamma/2\pi)\int_{-\pi+\delta}^{\pi} \mathrm{d}\phi\,[U_0(\delta) - (1-\phi/2\pi)\delta + \cos\phi]^{1/2}. \quad (7)$$

Figure 5 shows this analytic result for I_c^- as a function of δ. For $\gamma = 0.1, 0.2$ and 0.5, this yields $I_c^- = 0.30, 0.42$ and 0.79 respectively, compared with numerical values obtained from figure 2 of 0.30, 0.41 and 0.66. As expected, equation (7) yields good agreement with the simulation for small γ.

4. Discussion

We have demonstrated that quasi-particle resonances produce measureable changes in the I–V characteristic of weakly linked mesoscopic dots. In particular, for underdamped junctions, the critical current I_c^- is shifted to higher values and the linear portion of the characteristic no longer extrapolates to zero. Since the temperature at which the quasi-particle phase breaking length exceeds a system size is distinct from other characteristic temperatures, this change in the I–V curves may be used to identify the transition to mesoscopic dynamics. In obtaining the above results, we have examined only the effect of changing the current–phase relation in a standard RSJ model of a Josephson junction. This is a somewhat heuristic approach and ignores many details. For example the damping coefficient γ in equation (4) is conventionally identified with the electrical conductance of the junction, whereas in the mesoscopic limit, many inequivalent conductances can be defined [9–11]. For the future, it is should be possible to address such features through a first-principles derivation of a dynamical equation for the phase.

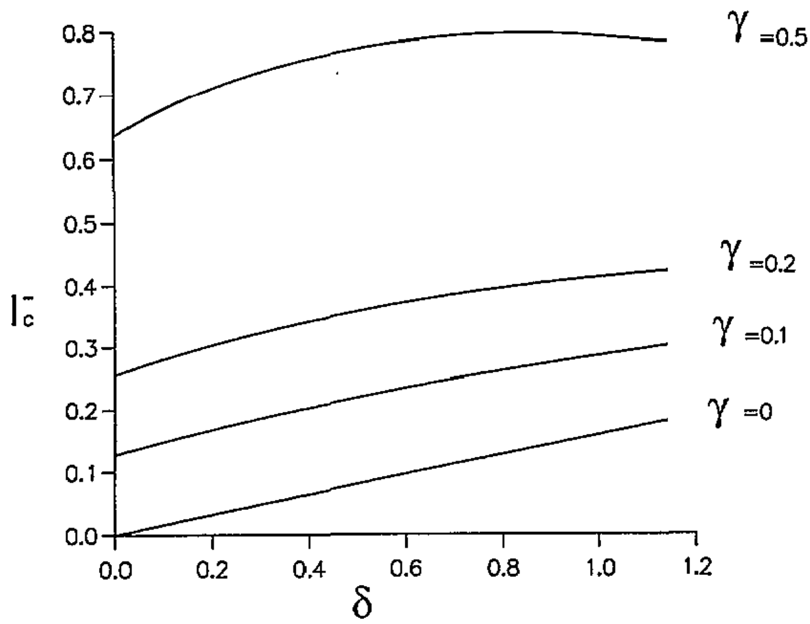

Figure 5. I_c^- as a function of δ for various values of γ, obtained by numerically evaluating the integrals on the right hand side of equation (7).

Acknowledgments

This work is supported by the SERC, the ISI, the ICTP, the EC and the MOD. It has benefited from useful conversations with M Leadbeater.

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